Karunya University
(Karunya Institute of Technology and Sciences)
(Declared as Deemed to be University under Sec.3 of the UGC Act, 1965)

End Semester Examinations - 2015-16 Even Semester - May 2016

15MA3019 Stochastic Processes

Set A

Time : 3 hrs
Total Marks: 100

1.

a. Show that the random process $X(t) = A\cos wt + B\sin wt$ is wide sense stationary if A and B are uncorrelated random variables with zero mean and same variance and w is a constant. (10marks)

b. Define random telegraph signal and prove that it is wide sense stationary. (10marks)

OR

2.

a. A Markov chain with three states has transition probability matrix $\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and initial distribution is (0.7,0.2,0.1)

Find i. $P(X_2 = 3)$

ii. $P(X_1=3/\ X_0=2\ )$

iii. $P(X_1=3, X_0=2)$

iii. $P(X_2=3, X_1=3, X_0=2)$

iv. $P(X_3=2\ \ X_2=3, X_1=3,\ X_0=2\ )$

b. Classify the states of the Markov Chain with transition probability matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{pmatrix}$ with state space 1,2,3.

3.

a. A housewife buys three kinds of cereals, A,B,C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys B. However if she buys B or C, the next week she is three times likely to buy A as the cereal. In the long run, how often does she buy each of the three cereal. (10marks)

b. A machine goes out of order, whenever a component fails. The failure of this part follows a poisson process with a mean rate of 1 per week. Find the probability that two weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (10marks)

OR

4. a. Derive probability law of poisson process. (15marks)

b. Define Birth Death process and state the postulates of Birth Death process. (5 marks)

5. a. Derive probability distribution of number of Renewals N(t) and find Mean of N(t). (10marks)

b. Particles reach a nuclear particle counter in accordance with Poisson Process with mean rate of 4 per minute, but the counter records only every third particle actually arriving. Find the probability that the number of particles recorded in first minute is equal to one. Find the probability that interval between two records is less than or equal to 1/2. (10marks)

OR

6.

a. The autocorrelation function of a stationary Random Process X(t) is given by

$$R(\tau)=\begin{cases}1-\frac{|\tau|}{T} & where\,|\tau|\le 1\\ 0 & otherwise\end{cases}$$

Find the power spectral density of X(t).

b. Find the first three autocorrelation coefficients of the moving average process

$$X_t = 1.6e_t - 0.8e_{t-1} + 0.4e_{t-2} - 0.2e_{t-3}$$

7. a. A supermarket has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4minutes, and if people arrive in a Poisson fashion at the rate of 10 perhour, (i) what is the probability of having to wait for service (ii)What is the expected percentage of idle time for each girl. (iii) Find the average queue length and the average number of units in the system. (iv) If a customer has to wait, what is the expected length of his waiting time. (10marks)

b. A car servicing station has 2 bays where servicing can be offered simultaneously. Due to space limitation, only four cars are accepted for servicing. The arrival pattern is Poisson with 12 cars perday. The service time in both the bays is exponentially distributed with 8cars per day per bay. (i) Find the average number of cars in the service station. (ii) Find the average number of cars waiting to be serviced (iii) Find the average time a car spends in the system. (10marks)

OR

8. a. An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. Find L_s, Lq, W_s, W_q, if the service time , (i) is constant and equal to 10minutes.(ii) follows uniform distribution between 8 and 12 minutes. (10marks)

b. In a two station service facility, queues are not allowed. Customers arrive at the facility at an average rate of 4 per hour. The server at each station serves at the rate of 5 customers per hour. If arrivals are Poisson and service times are exponential. Find (i) The probability that an arriving customer enters the system. (ii) effective arrival rate (iii) Expected number of customers in the system. (iv) Expected time a customer spends in the system. (10marks)

9. a. It is known that the reliability function for a critical solid state power unit for use in a communication satellite is R(t) = 10 / (10+t) , t>0 in years. How many units must be placed in parallel in order to achieve a reliability of 0.98 for 5 year operation. (10marks)

b. The time to failure in operating hours of an appliance has the hazard rate function $\lambda(t) = 0.003(t/500)^{0.5}$, t>0 in hours. (i) What is the reliability if the power unit must operate continuously for 50 hours. (ii) Determine the design life if a reliability of 0.9 is desired. (iii) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours of operation.

(10marks)

Wishing you All the Best
